



Rossmoyne Senior High School

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

SOLUTIONS

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	92	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (50 Marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

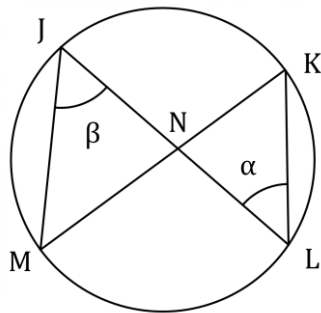
Question 1

(5 marks)

- (a) In the circle shown, chords JL and KM intersect at N , $\angle JMN = 42^\circ$ and $\angle JNK = 97^\circ$.

Determine the size of angles α and β .

(2 marks)

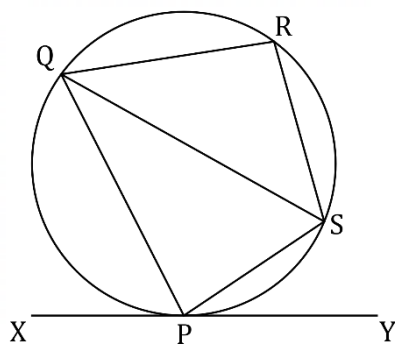


Solution
$\alpha = \angle JMN = 42^\circ$
$\beta = 97^\circ - 42^\circ = 55^\circ$
Specific behaviours
✓ value of α
✓ value of β

- (b) In the circle below, $PQRS$ is a cyclic quadrilateral and XY is a tangent to the circle at P .

Given that $\angle SPY = 38^\circ$ and $\angle QSP = 55^\circ$, determine the size of $\angle QRS$.

(3 marks)



Solution
Alternate segment: $\angle XPQ = \angle QSP = 55^\circ$
Angle on a line: $\angle QPS = 180^\circ - 38^\circ - 55^\circ = 87^\circ$
Opposite angles: $\angle QRS = 180^\circ - 87^\circ = 93^\circ$
Specific behaviours
✓ states $\angle XPQ$ or $\angle PQS$
✓ calculates $\angle QPS$
✓ calculates $\angle QRS$

Accept if values on diagram

Question 2

(6 marks)

Let $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j}$.(a) Determine $|3\mathbf{a} + 2\mathbf{b}|$.

(3 marks)

Solution
$3\mathbf{a} + 2\mathbf{b} = 3 \begin{pmatrix} 2 \\ -4 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 6 \\ -12 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ $= \begin{pmatrix} 8 \\ -6 \end{pmatrix}$ $ 3\mathbf{a} + 2\mathbf{b} = 10$
Specific behaviours
✓ calculates multiple ✓ calculates sum ✓ calculates magnitude

(b) Determine the vectors $\mathbf{p}$ and $\mathbf{q}$ given that $2\mathbf{a} - 3\mathbf{b} = \mathbf{p} - \mathbf{q}$ and $\mathbf{a} + \mathbf{b} = \mathbf{q} - 3\mathbf{p}$.

(3 marks)

Solution
Adding equations to eliminate $\mathbf{q}$: $3\mathbf{a} - 2\mathbf{b} = -2\mathbf{p}$ $\mathbf{p} = -\frac{3}{2} \left(\begin{pmatrix} 2 \\ -4 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right)$ $= \begin{pmatrix} -2 \\ 9 \end{pmatrix}$ $\mathbf{q} = 3\mathbf{p} + \mathbf{a} + \mathbf{b}$ $= 3 \begin{pmatrix} -2 \\ 9 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ 26 \end{pmatrix}$
Specific behaviours
✓ eliminates one vector ✓ calculates $\mathbf{p}$ ✓ calculates $\mathbf{q}$

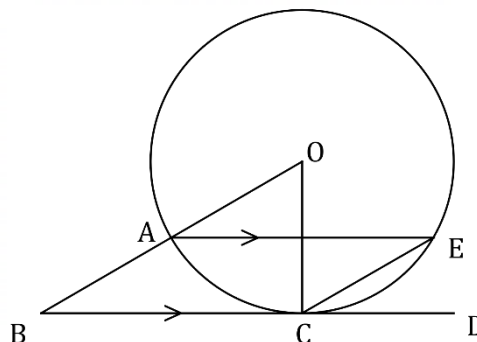
Question 4

(4 marks)

In the diagram, BD is a tangent at C to the circle with centre O , OB intersects the circle at A and chord AE is parallel to tangent BD .

Determine the size of $\angle ECD$ when the size of $\angle ABC = 38^\circ$.

Justify your answer.



Solution
Using angle between radius and tangent property: $\angle BOC = 90^\circ - 38^\circ = 52^\circ$
Using angle at centre and circumference on same arc property: $\angle AEC = \frac{1}{2}(52^\circ) = 26^\circ$
Using alternate angle property: $\angle ECD = \angle AEC = 26^\circ$
Specific behaviours
✓ angle at centre ✓ angle on circumference ✓ justifies both above properties ✓ correct angle

Question 4

(6 marks)

If there are a objects and b boxes, then the Pigeonhole principle says:

“If $a > b$, then at least one box must contain more than one object”

(a) Write the contrapositive of this statement.

(2 marks)

Solution	Specific behaviours	Point
If there are no boxes with more than one object, then $b > a$.	✓ Recognises that contrapositive statement implies $b > a$. ✓ Writes the correct contrapositive statement.	1.3.1

(b) (i) Prove that in any set of 6 distinct integers, it is possible to choose two of the integers such that their difference is divisible by 5.

(2 marks)

Solution	Specific behaviours	Point
When dividing the differences by 5, the remainders can be 0, 1, 2, 3, 4. There are 5 possible remainders, so from the 6 integers chosen, at least two of them will have a difference that will have the same remainder of 0 when divided by 5 and hence the difference is a multiple of 5.	✓ Uses remainders, or an equivalent mathematical explanation. ✓ Uses the pigeonhole principle to prove the statement.	1.1.6

(ii) Show that if eight of the first ten positive integers are selected at random, the selection contains at least three pairs whose sum is 11.

(2 marks)

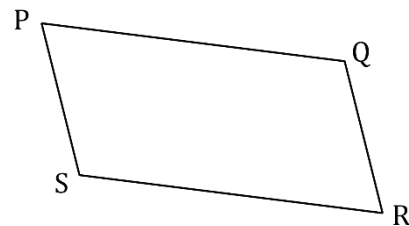
Solution	Specific behaviours	Point
The pairs of numbers 1 and 10, 2 and 9, 3 and 8, 4 and 7, 5 and 6 all add up to 11. If we pick one number from each of these pairs, we have only selected 5 integers. Hence the other 3 must be the other numbers in three of these pairs.	✓ Uses pairs, or an equivalent mathematical explanation. ✓ Uses the pigeonhole principle to prove the statement.	1.1.6

Question 5

(6 marks)

The diagram shows parallelogram $PQRS$.

A theorem states that the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of its sides.



- (a) Complete the following expression of the theorem using vector notation:

$$|\overrightarrow{PR}|^2 + |\overrightarrow{QS}|^2 =$$

Solution
$\dots = \overrightarrow{PQ} ^2 + \overrightarrow{QR} ^2 + \overrightarrow{SR} ^2 + \overrightarrow{PS} ^2$ Allow $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ or $2 \overrightarrow{PQ} ^2$, etc.
Specific behaviours
✓ correct expression and vector notation

(1 mark)

- (b) Letting $\overrightarrow{PQ} = \mathbf{q}$ and $\overrightarrow{PS} = \mathbf{s}$, use a vector method to prove the theorem.

(5 marks)

Solution
Note, for any vector $\mathbf{r}$: $ \mathbf{r} ^2 = \mathbf{r} \cdot \mathbf{r}$.
$ \begin{aligned} LHS &= \overrightarrow{PR} ^2 + \overrightarrow{QS} ^2 \\ &= \mathbf{q} + \mathbf{s} ^2 + \mathbf{q} - \mathbf{s} ^2 \quad \checkmark \\ &= (\mathbf{q} + \mathbf{s}) \cdot (\mathbf{q} + \mathbf{s}) + (\mathbf{q} - \mathbf{s}) \cdot (\mathbf{q} - \mathbf{s}) \quad \checkmark \text{ must have this step} \\ &= \mathbf{q} \cdot \mathbf{q} + \mathbf{q} \cdot \mathbf{s} + \mathbf{s} \cdot \mathbf{q} + \mathbf{s} \cdot \mathbf{s} + \mathbf{q} \cdot \mathbf{q} - \mathbf{q} \cdot \mathbf{s} - \mathbf{s} \cdot \mathbf{q} + \mathbf{s} \cdot \mathbf{s} \\ &= \mathbf{q} ^2 + \mathbf{s} ^2 + \mathbf{q} ^2 + \mathbf{s} ^2 \quad \checkmark \\ &= \overrightarrow{PQ} ^2 + \overrightarrow{QR} ^2 + \overrightarrow{SR} ^2 + \overrightarrow{PS} ^2 \quad \checkmark \\ &= RHS \end{aligned} $
Specific behaviours
✓ expresses $\overrightarrow{PR}$ and $\overrightarrow{QS}$ in terms of $\mathbf{q}$ and $\mathbf{s}$ ✓ uses scalar product to expand sums and differences ✓ simplifies scalar products as magnitudes ✓ expresses in terms of sides ✓ logical presentation of proof using correct vector notation

q or leaves
out • in lines
2-3 or 11

Question 6

(8 marks)

- (a) Determine the vector(s) that are parallel to $3\mathbf{i} - 4\mathbf{j}$ and have the same magnitude as $\mathbf{i} + 7\mathbf{j}$.

(4 marks)

Solution
Magnitudes: $ (3, -4) = 5, \quad (1, 7) = 5\sqrt{2}$ Unit vector in required direction: $\hat{\mathbf{a}} = \frac{1}{5}(3, -4)$ Hence vectors are: $\pm \frac{5\sqrt{2}}{5}(3, -4) = 3\sqrt{2}\mathbf{i} - 4\sqrt{2}\mathbf{j}, -3\sqrt{2}\mathbf{i} + 4\sqrt{2}\mathbf{j}$
Specific behaviours
✓ both magnitudes ✓ unit vector in required direction ✓ one required vector ✓ second vector in opposite direction

- (b) Two vectors are $4\mathbf{i} + (1 - \mu)\mathbf{j}$ and $\mathbf{i} + (2\mu + 5)\mathbf{j}$, where μ is a constant.

Determine the value(s) of μ so that the vectors are perpendicular.

(4 marks)

Solution
Require scalar product to be zero: $\begin{pmatrix} 4 \\ 1 - \mu \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2\mu + 5 \end{pmatrix} = 0$ $4 + (1 - \mu)(2\mu + 5) = 0$ $2\mu^2 + 3\mu - 9 = 0$ $(2\mu - 3)(\mu + 3) = 0$ $\mu = 1.5, \mu = -3$
Specific behaviours
✓ equates scalar product to zero ✓ calculates scalar product and expands ✓ simplifies and factorises ✓ states both values

Question 7

(8 marks)

- (a) Show that $4 \binom{5}{1} = 2 \binom{5}{2}$.

(2 marks)

Solution
$LHS = 4 \times 5 = 20, \quad RHS = 2 \times 10 = 20 \Rightarrow LHS = RHS$
Specific behaviours
✓ expresses LHS as product and evaluates ✓ expresses RHS as product and evaluates

- (b) Show that $(n - r) \binom{n}{r} = (r + 1) \binom{n}{r + 1}$ for all n and r that are positive integers, $n > r$.

(4 marks)

Solution
$ \begin{aligned} LHS &= (n - r) \binom{n}{r} \\ &= \frac{(n - r)n!}{(n - r)! r!} \\ &= \frac{n!}{(n - r - 1)! r!} \\ &= \frac{(r + 1)n!}{(n - [r + 1])! (r + 1)!} \\ &= (r + 1) \binom{n}{r + 1} \\ &= RHS \end{aligned} $
Specific behaviours
✓ expression using factorials for LHS ✓ divides fraction by $n - r$ ✓ multiplies fraction by $r + 1$ ✓ expresses as RHS

- (c) Hence, or otherwise, evaluate $\binom{65}{59}$, given that $\binom{65}{60} = 8\,259\,888$.

(2 marks)

Solution
$ \begin{aligned} (65 - 59) \binom{65}{59} &= (59 + 1) \binom{65}{59 + 1} \\ \binom{65}{59} &= 10 \times 8\,259\,888 \\ &= 82\,598\,880 \end{aligned} $
Specific behaviours
✓ indicates appropriate method ✓ correct value

Question 8

(7 marks)

Points P , Q and R have position vectors $(0, 5)$, $(-6, -5)$ and $(10, -1)$ respectively.

- (a) Determine the position vector of M , the midpoint of Q and R .

(1 mark)

Solution
$\overrightarrow{OM} = \left(\frac{-6 + 10}{2}, \frac{-5 - 1}{2} \right) = (2, -3)$
Specific behaviours
✓ calculates position vector

- (b) Determine the vectors $\overrightarrow{QR}$ and $\overrightarrow{PM}$.

(1 mark)

Solution
$\overrightarrow{QR} = (10, -1) - (-6, -5) = (16, 4)$
$\overrightarrow{PM} = (2, -3) - (0, 5) = (2, -8)$
Specific behaviours
✓ calculates vectors

- (c) Show that $\overrightarrow{QR}$ and $\overrightarrow{PM}$ are perpendicular.

(2 marks)

Solution
$\overrightarrow{QR} \cdot \overrightarrow{PM} = (16, 4) \cdot (2, -8) = 32 - 32 = 0$
Since scalar product is zero, then vectors are perpendicular.
Specific behaviours
✓ calculates scalar product
✓ interprets product

- (d) Hence, or otherwise, determine the area of triangle PQR .

(3 marks)

Solution
$\overrightarrow{QR}$ is base of triangle and $\overrightarrow{PM}$ is perpendicular height:
$ \begin{aligned} A &= \frac{1}{2} \overrightarrow{QR} \overrightarrow{PM} \\ &= \frac{1}{2} \times 4 (4, 1) \times 2 (1, -4) \\ &= 4 \times \sqrt{17} \times \sqrt{17} \\ &= 68 \text{ sq units} \end{aligned} $
Specific behaviours
✓ identifies base and perpendicular height ✓ calculates magnitudes ✓ calculates area

Supplementary page

Question number: _____

